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Notes on Bootstrap.

1) 1967 International School of Physics "Ettore Majorana" - Hadrons and their interactions - Notes from Closing lecture on "The present status of the fundamental particle interactions" by M. Gellmann. pp. 689 - 701.

I decided instead to speak about the problems we do not understand - In most of the changes that have occurred since last fall, I am not very well educated and many of them I heard of here, from Coleman, Glashow, Cabibbo and others - We know a great deal and we know very little. The paradox is that for each particular situation we have a fairly well developed formalism for eg (i) if we have to scatter electrons from positrons we use Quant. Electrodyn, (ii) for weak interactions we invoke the current x current picture, (iii) to describe cross-sections at high energies & angular distribution at small angles, we invoke diffraction scattering & Regge trajectories, (iv) to calculate soft pion emission we use current algebra and PCAC, (v) to calculate the spectrum of particle states we use the Quark-model and so on. We have the answers for all these solutions, and even if some of them are doubtful, a number of them are pretty good, but the relations among them are exceedingly obscure.

Why separate elementary microscopic phenomena into several departments? It seems to be a strange way for the things to be organised

(a) Gravitation with its own rule & its beautiful geometrical interpretation given by Einstein, which is elegant although not necessary for writing down the formalism

(b) Electromagnetism, which works extremely well in describing electromagnetic phenomena, and which bears only a superficial resemblance to gravity.

(c) The weak interactions, which have certain similarities to electromagnetism and great differences

(d) The strong interactions, associated with the hadrons

Perhaps between (a) & (c) there may be a new interaction, the Cronin-Fitch interaction, although this may not be a new thing but some different aspect of (c) itself or of (b) or of even (d).

p. 690 - Interesting remarks about quantum gravity (No plan ahead & no possibility in 50 or 100 years to test quantum aspects of gravity) - Also mentions that in Feynman's work of the last 11 years, one has of course to ignore the cosmological implications of gravity.

. 691 - If we had gravitons, I may say parenthetically we could do some amusing things. For eg. we can have a graviton making a neutrino-antineutrino pair [illegible] h

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Since the neutrino has energy, it must of course couple to gravity, and we can ask i this case a question which is otherwise unanswerable: is there a right-handed electron-neutrino? Since it is not coupled to weak interactions, which is the on place where we see neutrinos, there is no way to say, at present, whether there is a right-handed neutrino.

p. 691-2 - Remarks about obituaries about quantum electrodynamics all having proved false & its doing as well as ever. — Relation between electromagnetism and weak interactions,

p. 693 - Relation between weak interaction currents of leptons and hadrons, radiative correction; ignorance about the fundamentals — leptons themselves are sort of bothersome things; why do they have this curious spectrum? Two masses of ~~mas~~ nearly massless neutral things, and then two charged objects with different masses. Remember what Rabi said about the muon: who ordered that? We have absolutely no idea of what would go wrong with natural schemes if we took away the muon? — Particularly striking thing about leptons is their having spin 1/2 and we can get accurate results about them by using a treatment using Lagrangian field theory, such as electrodynamics. I lectured last spring at Cambridge Univ on current algebra and quarks, and it surprised me very much that Dirac liked both current algebra and quarks. He thought that quarks were marvellous; I could'nt understand what he liked about them, as they have many annoying properties; finally I found that he loved them because they have spin 1/2 .. ~~If~~ If it is really true that some of the symmetry properties of the hadrons can be

p. 694. described as if the hadrons were made up of quarks, and a crucial feature is spin 1/2, then there is something which closely ties leptons to hadrons.

However there is something we do not understand. Quarks may be a sort of lepton coupled to some special field which make them interact so strongly, or perhaps, they are purely illusory: a passing phase in our description, which ✓ will go away after a while, when we will learn how to use the bootstrap method and solve our equations without using quarks.

✓ Now suppose the hadron properties are explained by the Bootstrap; then we are faced with a situation which is completely different from the lepton case. There we have a small number of well-defined particles which act as elementary nd correctly described by Quantum electrodynamics, and here we have a system of highly emocratic hadrons which are all made up one of another. In either case there is a very eculiar situation

We heard this morning about the octet violation of SU_3 symmetry, and let us think ow many times the octet occurs in our physics today. We treat the hadrons with weak nd e.m. interactions as perturbations. Then we have the weak and e.m. currents fitting to SU_3. However we do not know if this is exactly true with the present experimental ccuracy, there might be some pieces in the weak currents which bring in terms ther than octets.

Actually SU_3 does'nt appear directly, if we just look at electromagnetism and the eak interaction, from them we only get a "twisted" SU_2 as Cabibbo showed us his morning. SU_3 comes only when we take the weak currents and divide them up into pieces ccording to strangeness which is conserved in strong interactions.

Another important fact is that the Strong interactions are approximately symmetrical under SU_3. According to the Bootstrap this has no particular significance (of course in the Bootstrap nothing has any significance, because everything is determined from self-consistency), but anyway this is quite striking – Refers to his formulation of this approximate symmetry

The symmetry violation in the strong interactions is also an octet, the eight-component of an octet. According again to the spontaneous symmetry violation in the Bootstrap it is implausible that it should be just octet; many people, including Dashen and Frautschi and others, have made it ~~possible~~ plausible that the splitting should be mainly octet, but it is very difficult in the Bootstrap to explain why it should be purely octet, and we don't indeed know that it is

.695 – One thing which amazes me is the lack of communication between people who work on the Bootstrap and people who do anything else. Because people who work on the Bootstrap talk in terms of something called S-matrix theory, which is supposed to be different from what everybody else does – Reviews briefly the connections between S-matrix theory, abstract field theory, Feynman diagrams etc...
So I don't see any difference; it seems to me that people are quarrelling only over terms s that everybody is working within the same frame-work (to be read again)

p. 696 – Seminars like the one this morning should be more frequent. I hope that in the near future there will be more investigations which are devoted to trying to xplain more deeply questions which are ~~unanswerable~~ unanswered, because we have the tendency to announce and publish only things that are complete — Next a discussion of CP-violation suggesting various possibilities for eg. existence of some particles which go into themselves under charge-conjugation. So there must be somewhere some new particles which such strange properties. There are so many particles which people tell us to find & we don't find for eg. quarks, intermediate bosons, mag. monopoles & above type of particles, and so on & so forth. What I suggested was that they might be the same particle, so that when we find one, we get all of them

p.697. other suggestions like an interaction very similar to weak interactions, but pertaining to hadrons only, that is purely non-leptonic, is a product of a scalar and a pseudo-scalar instead of vector & axial-vector, and is completely CP-violating ie is of $(1+i\gamma_5)$ type — About work in hadron physics

.698 ... so that when we pay all the money to get the machine in Serpukov, or those to be built in Illinois or somewhere in Europe, all that we get is a lack of simplicity; it is terribly unfair — discusses theory of dominant cut —

.699 I said a few words, during my series of lectures, on a quite technical subject, about the possible relations between the Bootstrap idea on the one hand and the Quark model on the other hand. Let me repeat them briefly. The hope was expressed that perhaps they are both right. Suppose the bootstrap method gives all or mostly all of the information of hadron dynamics (I am an agnostic, by the way, and whether the Bootstrap need be supplemented by certain other conditions), then how come that nucleons and mesons look somewhat as if they were made of quarks? Of course, one possibility, a terrible possibility, is that quarks exist and they themselves bootstrap. More likely, I think, is that quarks are mathematical, and the description of baryons and mesons in terms of quarks corresponds to a rough sketch of the symmetry properties of the solution of the Bootstrap equations

I had hopes to find the quark-model as a representation of current algebra, and then to be able to apply the Bootstrap conditions and see if there is some mass-formula, preferably agreeing with the expts, which would more or less achieve the Bootstrap conditions. However, the programme has so far not had sufficient success to prove this conjecture. What is certainly true is that these two systems of ideas are not supplementary to each other, because they are oo big. So either one of them is wrong, or both of them are wrong, or somehow they are ompatible

Structure of complex plane of A.M – mentions Mandelstam's unreadable paper!

.700 – Behaviour of Regge poles at zero energy. .. from this work it emerged that we have a unified description of two phenomena that have been noticed in the last few years ~~set~~ : the conspiracy of Regge trajectories and the Regge trajectories daughters. criticism of arguments against Reggeism – other things about the strongly interacting systems which we don't know – Looking back at list of interactions & relations; again mentions the Fitch – Cronin interaction)

.701 – I hope that during the next years we will find more occasions to reflect on the relations among all these ideas and more leisure to discuss them, and perhaps we can take some time of that time away from the time we spend on detailed calculations.

2) Proc. Int. Conf. 1967 (Univ. of Rochester)

Z = 0 & Bootstraps by F. Zachariasen, p. 687.

We should be able to phrase the bootstrap hypothesis of Chew & Frautschi [Phys. Rev. Lett. 7, 394 (1961)] in the language of Lagrangian field theory. The bootstrap idea was originally stated as the requirement that all strongly interacting particles lie on Regge trajectories ~~or~~ ie, that all S-matrix elements are analytic functions of the complex A.M, and that no Kronecker delta terms in the A.M. exist.

The bootstrap hypothesis, since it has always been phrased in the language of S-matrix theory, has necessarily suffered from the fact that this theory is not a fully developed theory, primarily in that not enough is understood about the singularity structure of many particle amplitudes. Hence use of Lagrangian field theory may be useful as the framework for the bootstrap idea – other advantages of this – $Z_{0\pi\pi}$ and Z_0 are the vertex and wave fn renormalisation constants – The simplest bootstrapped world we can imagine is one containing only one scalar particle & generalising from this we may state what the bootstrap hypothesis is in a field theory (i) First we hypothesize a world ie we assume a ~~system~~ given set of particles of specified space-time properties, (ii) write down the Lagrangian field theory sticking, for convenience, to all the Yukawa couplings permitted by the space-time conservation rules (iii) "solve" the field theory by whatever methods we can in terms of the coupling constants (one for each vertex) ~~and~~ the particle masses, (iv) ~~Among all~~ Set all the Z's (renormalisation constant one for each vertex and one for each particle) equal to 0 & there being just as many Z's as there are coupling constants and masses, the eqns Z = 0 eqns are solved for all these. If a solution exists, this determines all the parameters on which the already computed scattering amplitudes depend, and thereby we compute the bootstrapped solution. If no soln to the Z = 0 eqns exist, the hypothesised world is not a possible bootstrapped world – Bootstrap merely consists of adding the extra constraints Z = 0 to an otherwise pure field theory – Discusses number of difficulties of the procedures (i) to (iv) – Case of adding a photon ~~of~~ spin one & mass 0 to a bootstrapped world (with Z = 0) is shown not to involve any new equation (because of Ward's identity ?) & in this new world the value of the fine structure constant should be determined by the bootstrap conditions. Similarly it can be shown that a world with elementary leptons as well as an elementary photon (lepton not coupled only to γ) ~~also~~ has also the same number of bootstrap eqns as it has the parameters to be determined. The above argument does not show that a world containing but strongly interacting particles cannot exist, but if such a world did exist, some additional parameter would have to appear, or else one of the bootstrap eqns must be identically satisfied through some as yet not understood mechanism.

3) Article by Chew & Frautschi (Phys. Rev. Letters, 7, 1961, p. 394) – Does not at all mention the word "bootstrap". – The notion inherent in field theory (Lagrangian) that certain particles are fundamental is becoming less & less palatable as the no. of baryons & mesons ~~[illegible]~~ continues to increase as claimants as elementary particle – Sakata's suggestion of only p, n & Λ being elementary is arbitrary. Heisenberg's suggestion and the corresponding Feynman's criterion that a correct theory should not allow a decision as to which particles are elementary. The paper proposes a formulation of this criterion or principle within the S-matrix framework – The criterion proposed rests on the analytic structure of the S-matrix regarded as a simultaneous function of A.M and energy, quantities meaningful for arbitrary particle combinations – Reference to Regge's work that the S-matrix can be simultaneously continued into the complex energy and A.M. planes. For scattering by a superposition of Yukawa potentials, all poles are associated with bound states or resonances and may be viewed either in the E-plane for fixed J ~~and~~ or in the J-plane for fixed E. ~~Any pole~~ Feynman's principle

can be satisfied by postulating that all poles of the [illegible]; S-matrix and Regge [illegible] (status of Regge poles not clear) – other discussions re: field theory, maximal analyticity, distinguishing between Regge poles and elementary particle poles, "strips" in the Mandelstam diagram, low mass particles, approximately attractive forces.

4) Univ. of Colorado Series of Lectures in Th. Physics, 1965, p. 323 – Reggeised bootstraps by Bran Nath – Contains no general results of importance – Refers to an extensive bibliography "Lectures on Bootstraps" by F. Zachariasen, 1965 Pacific International Summer School in Physics, Honolulu, Hawaii (not available in the library).

5) Lectures in theoretical high energy physics – Edited by H. H. Aly (Beirut, Lebanon) 196 p. 51 – Bootstraps and their field quantisation by J. G. Taylor.

The purpose of the paper is to present a quantum field theory approach to bootstraps. By a bootstrap we mean here a dynamical system which contains no elementary particles but only composite ones; these composite particles "bootstrap" each other in the sense that they bind each other, to produce themselves as composites, through forces which arise from exchange of themselves. Such a bootstrap approach has the attractive possibility of determining the values of all coupling constants and masses in terms of one basic mass, these values being unique.

We use the field theory as an alternative to the on-mass-shell S-matrix approach because it enables many-particle intermediate states to be gathered together at once and dealt with by operator techniques. It is the purpose of this paper to present some interesting results obtained from the field-theoretic form of bootstrapped systems.

Every local field theory describing a system of particles commences with ~~bare~~ bare (or non-interacting) particles and clothes them to become physical particles by means of the interaction term in the Hamiltonian. Thus there will be two single particle states | bare > and | physical > for each type of particle. If we define $Z = |\langle \text{bare} | \text{physical} \rangle|^2$ being the probability of observing a bare physical particle in a physical state, then Z is the particle's wave function renormalisation constant. We set Z = 0 and the existence of a physical particle as the compositeness condition (for this particle) [Looks like Zachariasen's approach].

There has been a great deal of work recently attempting to derive the internal hadron symmetries of SU(3), SU(6) etc by means of bootstrap arguments – [Ref: Trieste, 1965 conference, Book II on high energy physics & elem. particles]

The simplest Yukawa coupling giving a non-trivial meson-nucleon bootstrap arises from the interaction $\bar{\psi}\gamma_5\gamma_\mu\psi\cdot\partial_\mu\phi$ and the resulting bootstrap eq^ns are

$$\psi = \lambda\gamma_5\gamma_\mu\psi\partial_\mu\phi$$
$$\phi = \lambda'\partial_\mu(\bar{\psi}\gamma_5\gamma_\mu\psi) \qquad \text{———} (a)$$

(a) is the PCAC model of the pseudoscalar meson – Some results trivial results having notion of "Kinks" & using field theoretic method

6) ~~High~~ Particle interactions at high energies – Scottish Universities Summer School 1966, edited by Priest & Vick (1967)

Art. 8. Quark models by E. J. Squires (Univ. of Durham)

The unitary symmetry classification of hadrons became relevant to physics when Gellmann & Ne'eman had the idea of putting the eight best known baryons & mesons into the 8-dimensional representation of SU(3). The resulting symmetry scheme has had considerable success in providing a phenomenological framework for describing experimental data on elementary particles. Regarding baryons as being made up of 3 quarks & mesons as being made of a quark-antiquark pair, we have, in terms of the dimensions of SU(3) representations:

For quarks $\qquad 3\otimes3\otimes3 = 10\oplus8\oplus8\oplus1$

For quark-antiquark pair $\qquad 3\otimes3 = 8\oplus1$

Corresponding to SU(6) rep^n we have: $6\otimes6\otimes6 = 70\oplus70\oplus56\oplus20$

& $\qquad 6\otimes\bar{6} = 35\oplus1$.

– Physical results rather than group theoretic approach used – Summarising the present situation so far there is no direct evidence for existence of quarks – Are quarks fundamental? Possibility of extending Chew's idea of nuclear democracy to quarks. To illustrate this we recall that, according to the reciprocal bootstrap idea [Ref. Chew, 1962, Phys. Rev. Lett. 9, 233] the existence of the nucleon and '33' resonance is a consequence of the existence of the pion i.e. they make themselves in a consistent way. In general, there is no objection in principle to the idea that quarks are self-consistently bootstrapped particles – A lot of unintelligible details and references.

p. 294 – The algebra associated with the vector and axial vector currents by L. A. Radicati

No time to discuss all types of "current algebra" – Discussion of SU(3) algebra associated with the hadron vector current (responsible for the electro-magnetic and by the semileptonic transitions with or without hypercharge conservation) on p. 31 one finds that what 'PCAC' means is partial conservation of the axial current

7) Chew's article in Phys. Rev. Lett. referred to in (6) – Reciprocal bootstrap relation of N and the (3,3) resonance

It has been proposed {Ref. to (3)} that within the frame work of the analytically continued S matrix all strongly interacting particles are composites held together by forces associated with "crossed channels" [Ref. in addition to (3) viz Phys. Rev. Lett. 8, 41 (1962); Phys. Rev. 124, 264 (1961) and Chew's book "The S-matrix theory of strong interactions"] Such a mechanism has been explored in some detail for the ρ meson, which, in a crude first approximation, appears as a composite of two pions mutually attracted by the exchange of a ρ ~~[illegible]~~ – The essence of the Chew-Low theory for the 3-3 resonance [Phys. Rev. 101, 1570 (1956)] is that in the first approximation this particle is a pion-nucleon composite, held together primarily by exchange of a nucleon. It is shown here that in the same sense and with the same degree of approx. verification, the nucleon is approximately a composite of a pion and a nucleon bound together to a large extent by exchange of a (3,3) (i.e. the reciprocal bootstrap) – Non-rel. calculation using Regge-type theory and the N/D language to prove the result (a lot of complex variable analysis) – It is reasonable to hope that a relativistic version of the bootstrap calculation outlined here, taking due account of the Regge asymptotic behaviour can be set up. If this goal can be reached the most striking characteristic of strong-interaction theory will have been demonstrated! It would then be almost certain, even without a detailed treatment of the strange particles that no arbitrary parameters can be tolerated.

8) Springer Tracts in Modern Physics Vol. 45 (1968) – The whole volume is devoted to Regge Poles in Particle Physics by Collins & Squires [Postscript, Jan 68

~~Chap VI – Bootstraps~~

~~Chap IV – The S-matrix.~~

Introduction – Generalisation of A.M. l to a complex variable (Watson, 1918, Sommerfeld 19 was found useful after work of Regge in ~~1961~~ 1959, which showed that for a wide class of potentials, the only singularities of the non-rel. ~~[illegible]~~ scattering amplitude in the l-plane were poles ~~with~~ which move with energy ie poles at positions $l = \alpha(s)$. These are now called Regge poles $\alpha(s)$ is called a Regge trajectory. The fact that Regge trajectories correspond to physical particles [or resonances] (Re α(t)) equals an integer for positive t, attracted attention of elementary particle physicists.

The Regge pole concept also permitted clarification of the distinction between a "bound state" which lies on a Regge trajectory, and an "elementary" particle which does not lie on a trajectory, but corresponds to a Kronecker δ-fn at the value of J equal to the spin of the particle. This gave a stimulus to the notion of "nuclear democracy" that all strongly interacting particles are bound states, and to the bootstrap hypothesis.

Shrinkage of popularity from 1962, and revival of interest since 1965 leading to the introduction of "conspirators" and "daughters". Cuts & fixed poles in the J-plane as well as the Reggeization of elementary particles have also been clarified in recent work.

The bible of all Regge theorists is the Bateman project [Erdelyi et al, 1953] with frequent use of "Higher transcendental functions" and "Tables of Integral transforms". Two mathematical points worth spelling out (i) notation about asymptotic behaviour of functions and (ii) frequent use of the word "analytic". When mathematicians say that a function is "analytic" in a ~~region~~ given region, they mean that it is free of singularities (holomorphic) in that region. Physicists tend to use the word much more loosely to mean reference to isolated singularities such as poles and branch points & not essential singularities, step fns, δ-fns or any other such pathological & non-smooth behaviour. Thus 'analytic' means 'analytic apart from isolated poles & branch points' & 'holomorphic' is reserved for the complete absence of singularities & 'meromorphic' for fns containing only poles in a given region.

(Chapter I - (1) The S-matrix — Is the analytic scattering matrix developed in dispersion theory — Properties of Regge poles can be formally demonstrated — It was within the framework of S-matrix theory that Regge poles were introduced into high energy physics — It is only in the relativistic theory that Regge poles are truly useful.

(2) Postulates of S-matrix theory

(i) Superposition principle of quant. mech
(ii) Existence of a unitary S-matrix
(iii) Lorentz invariance of S-matrix
(iv) Disconnectedness of S-matrix due to the finite range of the strong interaction forces

(V) Maximal analyticity of the first kind – i.e. that the only singularities of the S-matrix should be poles corresponding to stable or unstable particles, and the further singularities which are generated from those poles by unitarity

(vi) Maximal analyticity of second kind – i.e. the S-matrix should be continuable in A.M. throughout the complex A.M. plane, with only isolated singularities.

First four are relatively uncontroversial, while (V) & (Vi) are very controversial, but form the basis of dynamical S-matrix theory [Chew, 1964, 1965a, 1966]. It should be noted that (vi) follows from (V) at least in part, but it is convenient to treat it independently. (Vi) specially concerns Regge pole theory

(3) Bubble notation – Spin S-matrix theory

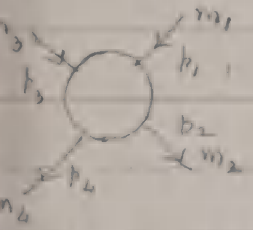
(4) Singularity structure of scattering amplitudes.

(5) Crossing and the TCP-Theorem – S-matrix is invariant under the combined operations of time-reversal, charge-conjugation and parity inversion

(6) The four-line connected part (the one given in Frazer) – Gives a Mandelstam diagram, showing the variables s, t, u. – diagram of physical regions for (a) equal mass scattering such as $\pi\pi \rightarrow \pi\pi$ (b) two different masses such as $\pi N \longrightarrow \pi N$

(7) Relation between scattering amplitude & measurable quantities.

(8) Singularities of the four-line connected part



uses relations like $s = M^2$, $s = (m_a + m_b)^2$ so that

~~[illegible]~~ [s = square of mass or energy].

– diagrams of singularities on the Mandelstam plot

(9) The two-particle discontinuity.

(10) Single variable dispersion relations – diagrams of (i) singularities in the complex t-plane at fixed s [t, s, u are all $\propto m^2$] & (ii) contour of integration in the complex t-plane

(11) The Mandelstam representation – using postulate (Vi) many ambiguities are resolved

So the basic difficulty of using an S-matrix theory based on postulates (i) to (V) is just this; that we do not know how much information, how many particle masses and couplings, if any, we need to put into the theory at the start. In non-rel. physics & perhaps also in electrodynamics, we can distinguish between the 'elementary' particles such as electron and proton, and the composite particles such as the hydrogen atom or the positronium (?), but with strong interactions this distinction is harder, perhaps,

(*) In $A^{\pm}(s, t)$ one gets Regge poles and also Regge cuts. The former also found in potential scattering & the cuts are to be expected in relativistic scattering. The poles in the l-[illegible]

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impossible to make. But evidently the answer to the problem is bound up with the A. P. of [illegible] of the S-matrix treated in the next chapter.

(11) The elastic double spectral function

Chap. II - Partial wave amplitudes and Regge poles

(1) Partial wave amplitude - Defn: A partial wave amplitude in the s-channel is

$$A_l(s) = \frac{1}{16\pi} \cdot \frac{1}{2}\int_{-1}^{1} dz_s\, P_l(z_s)\, A[s, t(z_s, s)] \quad \text{for } l = 0, 1, 2, \ldots$$

where the third factor in the integral is the amplitude $A(s, t)$ and the second factor is the Legendre polynomial of the first kind of order l.

(2) Amplitudes of definite signature - amplitudes having only right-hand singularities

(3) The Froissart - Gribov projection - Defn for $A_l^{\pm}(s)$

$$\left.\begin{aligned} A_l^{+}(s) &= A_l(s) \text{ for even } l \\ A_l^{-}(s) &= A_l(s) \text{ for odd } l \end{aligned}\right\}$$

give the "physical" values of l for the partial wave-amplitudes of definite signature

(4) Singularities of partial wave amplitudes - Diagrams showing cuts of the $\pi\pi$ and πN partial wave amplitudes

(5) Unitarity of partial-wave amplitudes and phase shifts

(6) Asymptotic behaviour and the projection of (3)

(7) The Froissart bound - Diagram of Lehmann ellipse

(8) Analytic continuation of AM - Uses Carlson's theorem (Ref. Titchmarsh, 1939, p. 186 for proof, viz. If $f(z)$ is regular and of the form $O(e^{K|z|})$ where $K < \pi$, for $\mathrm{Re}(z) \geq 0$, and $f(z) = 0$ for $z = 0, 1, 2, 3, \ldots$, then $f(z) = 0$ identically

(9) The Sommerfeld - Watson transform - 4 diagrams showing contours in the complex A.M. plane - use of hypergeometric fns - This transform also gives another way of seeing why Carlson's theorem gives a unique interpolation between the integers (*)

(10) Regge poles - Figures of sorts of trajectories found in potential scattering and which one can anticipate in particle physics - "Signature factor" appearing in $A(s, t)$ - Contribution of a Regge pole to a partial wave gives

$$A_l^{\pm}(s) = \frac{[2\alpha(s) + 1]\,\beta(s)}{[l - \alpha(s)][\alpha(s) + l + 1]}$$

Maximal analyticity of the second kind put all the particles on a similar footing, all are composites, the stable nucleon just as much as the unstable N^* or the stable deuteron which has always been considered composite [Refs to Chew & Frautschi, Gellmann & Zachariasen]

The existence or non-existence of elementary (non composite) particles is which can give rise to poles of $A^{\pm}(s,t)$ in the s-plane at values of $s = s_R$ such $\alpha_i(s_R) =$ an integer, since $\sin[\pi\alpha_i(s)] \to 0$ as $s \to s_R$. The fn $\alpha_i(s)$ is a "Regge trajectory". The [illegible] requires that $\alpha_i(s) \leqq 1$ for $s \leqq 0$, for all trajectories

This seems to be intimately connected with the validity of postulate (VI). It would seem that if we can demonstrate that all the particles do lie on Regge trajectories and postulate (VI) is valid, then no particle can be elementary

(1) Singularities of partial wave amplitudes for non-integer l.

(2) The Mandelstam - Sommerfeld - Watson transform

Chap III - Regge poles and the Mandelstam representation

(1) Some properties of the Regge functions - unitarity demands that $\mathrm{Im}\{\alpha(s)\}$ shall be +ve, so that $\alpha(s)$ is a Herglotz function

(2) Regge poles and Mandelstam representation

(3) The Chew - Jones Regge pole representation

(4) The Khuri - Jones "

(5) The Khuri power-series expansion - Hypothesis of Regge poles has been tied to the Legendre fn partial wave expansion & this results in some difficulties in that the scattering angle z_s is a fn of both s & t, while we are interested only in asymptotic properties of the amplitudes in s or t rather than z_s. - Khuri "satellites" - The complication of unequal mass kinematics have led some writers to doubt the potency of the Regge plane in particle physics

(6) The problems of unequal mass kinematics -

(7) "Daughter" trajectories - Regge asymptotic behaviour applies even at $s = 0$ where the Regge pole term itself becomes undefined. Using the expression for $A^{\pm}(s,t)$ obtained by using the Khuri power series expansion, and considering the $\sum_{n=0}^{\infty}$ & ~~writing out the~~ other Σ term, we have by writing out the first terms of the sum over n

$$-8\pi \frac{\gamma_0(s)[2\alpha_0(s)+1]}{(4)^{\alpha_0(s)}\cos\pi\alpha_0(s)} \cdot \frac{\Gamma[-\alpha_0(s)]}{\Gamma[-2\alpha_0(s)]} \left\{ (-t)^{\alpha_0(s)} - \frac{\alpha_0(s)}{2} 4q_s^2 (-t)^{\alpha_0(s)-1} + \cdots \right\}$$

The second term which has a simple pole in s at $s = 0$, could be cancelled by the leading term of a second Regge pole $\alpha_1(s)$ provided that the residue of this pole $\gamma_1(s)$ has a pole at $s = 0$ and $\alpha_1(0) = \alpha_0(0) - 1$. Explicitly we require that

$$\gamma_1(s) = \frac{\gamma_0(0)[2\alpha_0(0)+1]\Delta^2}{4s} + \text{regular part}$$

near $s = 0$. We ~~should also require a~~ third trajectory $\alpha_2(s)$ such that $\alpha_2(0) = \alpha_0(0) - 2$, whose residue has a double pole at $s = 0$, so that its leading term will cancel the third term of α_0 and the second term of α_1, and so on. An ∞ number of trajectories with every more singular residues is required. $\alpha_1(s)$ is known as the "first ~~traj~~ daughter", $\alpha_2(s)$ the "second daughter" & so on

1.

Dispersion relations – Scottish Univ. Summer School, 1960

p. 167 – Double dispersion relations and unitarity as the basis for a dynamical theory of strong interactions by Geoffrey. F. Chew.

The Introduction – The discovery by Mandelstam in 1958 of a prescription for extending collision amplitudes into the complex plane, as functions simultaneously of energy and momentum-transfer has brought Heisenberg's 1943 idea of a dynamical S-matrix theory close to realisation – In 1955 Chew & Low showed for the static model of the pion-nucleon interaction that one was dealing with singularities of the scattering amplitude in unphysical regions, and that a knowledge of the location and strength of these singularities was probably sufficient to determine the S-matrix – The general principle emerging from the work of the past five years based on field theory is that the S-matrix is the boundary value of an analytic fn of momentum variables, satisfying the substitution law [viz that a single analytic fn of two variables, $A(s_1, s_2)$ describes all three channels corresponding to a given diagram, the channel being selected merely by announcing the values of the variables] & with only those singularities required by unitarity. This is the notion of "maximal analyticity". Consistent with Lorentz invariance and unitarity the only prescription for extending 2-particle S-matrix elements into the complex plane, consistent with the substitution law, is that given by Mandelstam. If the condition of max. analyticity be fully formulated, it would be a promising candidate to replace the framework of field theory – Such a point of view has been expressed forcefully by Landau who feels that any further work with field theory is a waste of time. I concur in the belief that the field theory is inconsistent and will eventually die, but am impressed by the many apparently valid general principles (such as the substitution law) discovered by field theory by using dubious methods. The conclusion is that analyticity and the substitution law together with Lorentz invariance & unitarity are sufficient to determine the S-matrix

The other parts are better expressed in the Springer Tract No. 45

The role of daughters can also be shown in the Chew-Jones ref? – Freedman & Wang tried to give additional supporting evidence for daughters in their paper based on the properties of the Bethe-Salpeter equation, which is well-known to have a relation to Regge poles ~~theory~~ in perturbation theory (see Chap VII). They showed that daughters might follow from the 4-dimensional symmetry of the Bethe-Salpeter equation at $s = 0$. Recently these extra symmetries at $s = 0$ have been discussed from the more general point of view of invariance under the group O(4) which also involves questions about "conspiracies".

(8) Some experimental consequences – Most trajectories are too low-lying for their daughters to be observed, and this is also true of the better behaved second daughters of all trajectories. As G-parity also forbids most of the more straightforward decays of the boson daughters, it is not particularly surprising that they have not been directly observed – At present time the only "evidence" for daughters is purely theoretical.

Chap IV – Spin:

(1) Partial wave expansion for particles with spin – Extension to case of spin $\neq 0$. The theory can be developed in an elegant way in full generality – Extensive literature on the extension of Regge-pole ideas to particles with spin – Relativistic situation studied by Gell-Mann et al. – helicity formalism of Jacobson-Wick (as in Frazer's book) – use of "Reggeizing" or using "invariant amplitudes" rather than helicity amplitudes – Jacob and Wick's helicity formalism used in this chapter.

(2) Properties of $d^J_{\lambda\mu}(\theta)$ and $e^J_{\lambda\mu}(\theta)$.

(3) Kinematical singularities and single variable dispersion relations.

(4) Generalised Froissart – Gribov projection.

(5) The Sommerfeld – Watson Transform

(6) Kinematical constraints on the residue functions. – The natural way to obtain a zero (which at $s = 0$ in a t-channelled amplitude appears explicitly in the t-channel partial-wave expansion, but does not show up as a zero in a particular s-channel helicity amplitude) of this type in the amplitude is to require that the residue of any contributing trajectory also has the zero. In some cases, such a procedure may be incorrect when the zero does not correspond to a branch point and that when it arises from two or more trajectories whose contributions to the amplitudes in question cancel each other at $s = 0$. This possibility is referred to

as a "Conspiracy". – ~~Header~~ Leader [1967] has ~~shown~~ analysed in detail the form of the crossing matrix (Trueman & Wick) constraint for general spins & masses, and distinguishes three possible ways in which they might be satisfied viz (i) 'Evasion' in which the residues have the appropriate zeros & no compensating trajectories are required, (ii) "conspiracy in which a finite number of trajectories having different quantum numbers are so arranged that they satisfy the constraints provided they are all included, and (iii) "daughters" ~~where~~ which are an ∞-set of trajectories having the same quantum number.

(7) Parity – Since this is good quantum number for strong interactions it is convenient in many applications of Regge pole theory

(8) The Pomeranchon contribution – The evidence that total cross-sections tend to constants led to the idea of a trajectory which passes through $s = 0$ at the maximal position allowed by this condition ie $\alpha(0) = 1$; this is the so-called Pomeranchon trajectory

(9) Application of Group-theoretic methods to Regge Theory – Long and very interesting section to be looked into later on (if possible) – Talks of a single Lorentz pole whose existence predicts existence of daughters and conspirators – Also talks of the Bethe-Salpeter equation proving so useful for generating daughters & also other work re. B.S. eqn like "abnormal solns", also using this eqn to show that daughter trajectories behave in an extremely different way from the parent trajectory –

Chap. V – The nature of J-plane singularities.

(1) The analytic continuation of unitarity.

(2) Moving singularities in the J-plane. (Ref. to Drummond re. potential models involving ~~(3)~~ three particle systems, also extensible to relativistic scattering processes)

(3) Cuts in the complex angular momentum plane. (4) Fixed J-plane poles.

(5) Singularities in the Froissart-Gribov integral and superconvergence relations. –

General condition for the vanishing of the residue can be obtained as

$$\int_{t_0}^{\infty} D_t^{\pm}(s,t)\, dz_s = 0 \qquad \left[D_t^{\pm}(s,t) \text{ is the discontinuity in } t \text{ of } A^{\pm}(s,t) \right]$$

A relation of this type is called a "~~[illegible]~~ superconvergence relation".

– Reference also to usual Feynman rule (3)

6. Gribov-Pomeranchuk singularities. – Talks of sense-nonsense decoupling not necessarily occurring at wrong signature amplitudes, & hence that a given trajectory does not necessarily have to "choose" "sense" or "non-sense" when it passes through a wrong signature value of J – In Chap. IV (2) "sense-nonsense

amplitudes are defined as those corresponding to the scattering from a possible physical state to an impossible physical state (see article by Gellmann – Proc. Conf. of [illegible] on high energy physics)

Chapter VI. Bootstraps

(1) The Bootstrap Hypothesis – Maximal analyticity of the 1st kind tells us that a knowledge of the poles determines the double spectral function [The Mandelstam repn. for the four-line connected part gives

$$~~A_s~~\ A(s,t) = \frac{1}{\pi^2}\iint^{\infty}\frac{\rho_{st}(s'',t')}{(s''-s)(t'-t)}\,ds''dt' + \frac{1}{\pi^2}\iint^{\infty}\frac{\rho_{su}(s'',u')}{(s''-s)(u'-u)}\,ds''du'$$

$$+\frac{1}{\pi^2}\iint^{\infty}\frac{\rho_{tu}(t',u'')}{(t'-t)(u''-u)}\,dt'du''$$

Also $\rho_{st} = -\frac{1}{4}\left\{A(s_+,t_+) + A(s_-,t_-) - A(s_-,t_+) - A(s_+,t_-)\right\}$

and the ρ_{st}, ρ_{su} and ρ_{tu} are called the double spectral functions] completely, and maximal analyticity of the second kind tells us that a knowledge of the double spectral functions enables us to determine the poles completely. Evidently there is a very strong self-consistency condition here. For if one were to add some arbitrary pole to an amplitude, one would generate, via maximal analyticity of the first kind, a whole new set of singularities, a whole new set of contributions to the double spectral function. And these, because of max. analyticity of the second kind, would probably imply the presence of further sets of poles which would then imply further contributions to the double spectral ~~fn~~ functions and so on ad infinitum, unless at some point self-consistency were reached and the actual set of poles present in the amplitude implied the presence of just that set and no others.

The question then arises as to what set or sets of particles can satisfy this self-consistency condition. Clearly, if max. analyticity of the first & second kinds are true in the real world of strongly interacting particles, then the actual particles which have been discovered must be part of such a set. There are probably an ∞-number of particles in this set, though many are so unstable as to be undetectable. Are there, however, completely different sets of hypothetical particles which would satisfy our postulates? The conditions are so stringent that this seems unlikely, though we have no way of ruling it out rigorously at present. The unlikelihood has led to the proposal [See for instance Chew, 1962, 6, Rev. Mod. Phys, 34, 394] of the "bootstrap" hypothesis. We can express it as follows:-

The Bootstrap hypothesis – The only set of particles (poles) consistent with the principles of maximum analyticity of the first and second kinds, is the actual set of strongly interacting particles found in nature.

The word "bootstrap", of course, derives from the proverbially impossible feat of "pulling oneself up by one's own bootstraps", which the self-consistency requirement (perhaps unfortunately) suggests. Originally the idea was that a single particle might be approximately self-consistent in some amplitude so bootstrapping itself (eg. the ρ in the π-π scattering, but strictly all the particles should be included in a global bootstrapping operation.

It should be stressed that this is not a new postulate, but our hypothesis as to the consequences of our six postulates. Unfortunately, solns of the unitarity eqns involve soln of ∞ sets of coupled non-linear, singular, integral equations, so the hypothesis is impossible to test as a whole. To test it one would have to solve the entire strong reaction problem in one fell swoop. Instead, many attempts have been made, with varying degrees of success, to demonstrate that some small subset of the particles is approximately self-consistent, within some subset of the equations which is believed to be approximately decoupled from the rest. These are called "bootstrap calculations". With so many approximations, no test can ever be crucial, but a sufficiently large number of partial successes have been achieved to make the more optimistic feel that the hypothesis may be true.

Chapter devoted to ~~(i)~~ bootstrap calculations using the two principal techniques the (i) the N/D method for partial wave amplitudes, and (ii) the Mandelstam iteration.

Also concentrating on those types calculation which bring out the importance of Regge poles in bootstrap dynamics.

Remarks re. interdependence of the six ~~post~~ postulates, & in particular of the interrelation between (V) & (Vi) – ~~Recent work~~ We have seen that (V) leads to (Vi) wherever the Froissart – Gribov projection is defined ie for $\ell \geq \ell_M(s)$. Also the Froissart bound, required by cross-channel unitarity demands that $\ell_M(s) \leq 1$ for $s \leq 0$ – Recent work extending these limits has shown that if a scattering amplitude has a purely elastic region in two channels, and if the double spectral functions are known exactly in an arbitrarily small strip of one elastic region, then the subtractions (in the S- and P-waves) are completely determined, and there is no arbitrariness in the amplitude. Much work still remains to be done on these matters, but it seems perfectly possible that eventually (vi) will be deduced from (v). If it is, this will greatly strengthen the bootstrap hypothesis which will then become just the statement that the only set of particles consistent with (v) is the actual set of strongly interacting particles found in nature.

(2) The N/D equations. – These eqns introduced by Chew & Mandelstam [Phys. Rev. 119, 476 (1960)] are the most commonly employed for bootstrap calculations. They have in fact, been likened in importance to the Schrodinger equation of non-rel. quantum mechanics – impossible to follow the detailed mathematical calculations – only the eqns are integral equations – Consider the amplitude $A(S,t)$ for the scattering of two spinless particles discussed in Chaps I & II. The s-channel reduced partial-wave amplitude ~~is given by~~ which is

$$B_\ell^{\pm}(s) = \frac{e^{i\delta_\ell(s)}\sin\delta_\ell(s)}{P_\ell(s)} \qquad (1)$$

is given by ~~$= \frac{1}{16\pi^2}\int$~~ ~~This is easier to~~ To solve the eqn it is simpler if we first linearise it by making the decomposition

$$B_\ell^{\pm}(s) = \frac{N_\ell(s)}{D_\ell(s)} \qquad (2)$$

(*)

where the numerator has the left hand cuts & the denominator the right hand cuts of $D_\ell^{\pm}(s)$. The conventional normalisation condns are $N_\ell(s) \to 0$ & $D_\ell(s) \to 1$ as $s \to \infty$. Use of the Theorem of Wiener & Hopf [Titchmarsh (1937), p. 339] leads to the dispersion relation for $D_\ell(s)$ viz

$$D_\ell(s) = 1 - \frac{1}{\pi}\int_{s_0}^{\infty} \frac{P_\ell(s')\,N_\ell(s')}{s'-s}\,ds' + \sum_{i=1}^{P_\ell} \frac{r_{i\ell}}{s-s_{i\ell}}$$

where residues of the poles are taken $r_{i\ell}$. Since we can specify $r_{i\ell}$ & $s_{i\ell}$ arbitrarily it is evident that $D_\ell(s)$ is not completely determined by the input $D_\ell^{L}(s)$. This ambiguity is known as the CDD-ambiguity (after Castillejo, Dalitz and Dyson) and the poles are called CDD-poles. If there were CDD poles they would correspond to elementary ~~particles~~ particles since we can show (vi) is satisfied.

Assuming there are no CDD poles, we can write following dispersion relations for $N_\ell(s)$ and $D_\ell(s)$

* This leads to $\mathrm{Im}\{N_\ell(s)\} = b_\ell(s)D_\ell(s)\ [s < s_L]$; $b_\ell(s) = \mathrm{Im}\{B_\ell^{\pm}(s)\} - \mathrm{Im}\{D_\ell(s)\}$

not able to bootstrap itself

(5) The reciprocal N, N* bootstrap – It appears likely that N is really a CDD pole in the π-N channel & is bound in some other channel or channels. Problem not fully solved

(6) The new form of the strip approximation (Chew & Jones, 1964)

(7) The Mandelstam iteration – This provides the relativistic equivalent of scattering to the Born-series in non-relativistic scattering – is a modification of the "old" form of the strip approximation of Chew & Frautschi (1961, a) – Work of calculation forthcoming

(8) Some problems. – (i) New approaches to the strip approximation problem

(ii) Considering the quark model, the mass particles of two or three quark systems would be CDD poles in the amplitudes we are looking for. This presumably puts the main weight of the double spectral function out at $t \approx 4M_Q^2$ (M_Q = quark mass) & makes the sharp fall of the forward & backward peaks very hard to understand. One would expect $A(s,t) \sim \frac{1}{t-4M_Q^2}$

(iii) Problems of existence of internal isospin symmetry SU(2), the higher symmetry, including strangeness, SU(3), which is broken in such a regular manner, and the conjectured SU(6) dynamical symmetries. Much work done & remains to be done. But Regge poles feature hardly at all in these accounts.

Despite the problems which we have found the bootstrap concept retains its appeal, partly because it is clear that, if our analyticity postulates are correct, something of the sort has got to work, and partly because so many qualitative results support it. It is evident that the simple dynamical models which have been tried so far will need considerable refinement and sophistication before their suggestive features can become established facts. But the main point, which has been particularly emphasised by Chew (1965, a, High energy Physics; the 1965 Les Houches Lectures, Gordon & Breach) is that because the S-matrix theory is so closely related to the experimental numbers which it is intended to explain, it will continue to develop, providing that we go on incorporating into it as much of our experimental information as possible. We shall see in Chap. VIII that Regge poles summarise a tremendous amount of such information, and it will certainly be necessary to include them in any future dynamical scheme.

Chapter VII – Perturbation theory & Elem. particles

Chap VIII – A survey of the expt. situation – Quark model for baryons

Since any curve is sufficiently linear if we choose a sufficiently small part of it, a natural explanation of this apparent linearity is that the scale of the dynamics is much larger than the few GeV over which trajectories have been observed. An obvious, if highly speculative example of such a theory is the quark model of Gell-mann. For our purposes here the essential property of quarks is that they are heavy ($M_q \gtrsim 5$ GeV) and that mesons are made of quark and quark-pair bound states, and baryons as bound states of three quarks. The "potentials" between the quarks required to produce the necessary binding energies must be very deep. The trajectories would turn over eventually, but would be expected to pass through a large no. of bound states (i.e. bound w.r.t. the $q\bar{q}$ threshold) first.

An additional feature of this model is that explains the lack of any "signature" effect, at least for the mesons. To see this we note that the direct force between a q-$\bar{q}$ pair can be caused by the exchange of mesons (note that these give a potential of the correct range). However, the exchange force arises from u-channel exchanges and these have the quantum numbers of a qq pair (Baryon number 2/3). It is known that there cannot be such a particle with mass less than about 5 GeV, so that the exchange force is of extremely short range, and its small effect is therefore easily understood. The lack of exchange force, leading to overlapping trajectories of opposite signature, is sometimes referred to as "exchange degeneracy".

We note that this behaviour in the quark model is in striking contrast to the situation in conventional bootstrap theories of type described in Chap VI. In these theories the ρ is assumed to be predominantly a ππ-state, whereas the A_2 cannot contain any ππ contribution ($A_2 \to \pi\pi$ is forbidden by G parity) so that A_2 and ρ trajectories have completely different origins, and their degeneracy is apparently an accident.

However the quark model also has its difficulties – want of 4 or 5 q [illegible] states

We note that the quark model of high energy scattering is not inconsistent with the Regge-pole model and it might be profitable to combine them by Reggeizing the qq and $q\bar{q}$ amplitudes

Later work on Bootstraps (i) Finite energy sum rules of semi-bootstrap schemes have been discussed (1967). ii) Difficulty of reconciling straight Regge trajectories with a bootstrap of the type described in Chap VI have been emphasised ([illegible], 1968).

Abbe, Hams & Nath trying to obtain a self-consistent Reggeized ρ-meson in the method of approach used here (applying the Watson-Sommerfeld transform). It is an involved computational problem, but our preliminary results are interesting. We certainly obtain a minimum — remember that this need not necessarily have come out. In fact, if we fix the mass of the ρ-meson, the width turns out to be in quite good agreement with experiment. In the N/D calculation with elementary ρ exchange, the width comes out too large.

(1) Further remarks of Gell-Mann from the closing lecture

I am unable personally to perceive the difference. Let me review for a moment how I see this problem. It seems to me that everything we do in particle physics, all the serious theoretical work, is inside the same framework. And this framework may be described in several different ways; the oldest way to describe it is to assume that there is at least one local operator, such that commutator with itself at distinct spacelike points gives zero (that means we have relativity & causality & so forth). All such things, when expressed in rigorous terms, we can call abstract field theory. Within the game of abstract field theory everything seems to fit, as far as I can see; there are people who proved dispersion relations and such things from abstract field theory, and very slowly they come up with the same list of ~~diagrams~~ dispersion relations that people had gotten from looking at old Feynman diagrams. At the same time these dispersion relations can be restricted to the "mass-shell"; you then get a set of formulae which represent analyticity of amplitudes for various numbers of particles on the "mass shell". These analyticity properties are chosen as postulates by certain people, who then say they are doing S-matrix theory. Presumably what they mean is that in their formalism there is no equivalent to the original local operator. But why not? All we have to do is to take over from the field theory the linear homogeneous dispersion relations obeyed by quantities defined off the mass-shell, like local operators within the S-matrix theory; then, by using the S-matrix elements as coefficients you can solve these equations and get formulae for those local quantities; and then you have currents, stress energy tensor, and so on, I should say, adjoined. to the S-matrix theory. In fact, you can show, as Dashen and Frautschi do, that if you commute two such things you get another local thing, defined in this way in the S-matrix theory. So I don't see any difference;

— Also refers to Teller's results found in the Heidelberg Conference Proceedings

115• Lectures on Th. Physics : Vol. 9B – Univ of Colorado, Vol IX . B. (1967) – High energy and particle physics

p. 281. Bootstrapping, static models & strong coupling by C. Goebel (Wisconsin)

(A) Brief sketch of solns of the Cutkosky bootstrap eqns (CBE)

[Phys. Rev. 131, 1888 (1963) & ibid 135, B1445 (1964)] – By CBE is meant

$$\sum_{B,C} g^{A}_{BC}\, g^{BC}_{A'} = \text{const} \times \delta_{AA'} \qquad (1)$$

$$\sum_{a,b,c} g_{Ab}{}^{c}\, g_{Bc}{}^{a}\, g_{Ca}{}^{b} = \text{Const.} \times g_{ABC} \qquad (2)$$

where g_{ABC} is the trilinear coupling constant of the three particles A, B, C, lower case letters refer to exchanged (virtual) particles. The constants on the R.H.S. of eqns (1) & (2) are independent of A and of A, B, C resp. These bootstrap eqns result from consideration of the Bethe-Salpeter wave eqn in the 'ladder' approximation, taking the particles to have equal masses.

A soln of the CBE eqns (1) & (2) is

$$g_{ABC} = \sum_{r} \begin{pmatrix} J & J & J \\ A & B & C \end{pmatrix}_{r} \qquad (3)$$

if the particles comprise the states of a representation "J" of a compact Lie group ("symmetry group") and the couplings are 3-J symbols (Clebsch-Gordan coeffs) of that group. More precisely, they are in general a linear combination of 3J symbols being summed over the "coupling index" r i.e. over the three different ways that J × J × J can be coupled into a singlet. The satisfaction of (1) & (2) by (3) is a consequence of the well-known properties of 3J-symbols, (1) being their orthonormality, and (2) being an example of the Wigner-Eckart theorem.

Use of Racah diagrams for geometrical illustration instead of eqns

(B) Another soln of (1) & (2) is one in which the non-vanishing vertices are between two particles, belonging to possibly distinct multiplets J_1, J_2 and a third particle which belongs to the (reducible) multiplet $J_1 \times J_2$.

(C) A third type of soln is provided by the rigid rotator class of static models strong coupling solutions. Here the particles are of two different kinds, 'mesons' & 'baryons', the latter being ∞ly massive scatterers in the strict

Static's model. The vertices are of the form baryon – baryon – meson. Actually eqn (1) is not going to be strictly satisfied, since the constant on the R.H.S. will be different for mesons & baryons.

Mention of another class of strong coupling solns: the "harmonic oscillator" type, which do not satisfy eqn (1) or 2)

Coupling limits & bootstrap eqn in static models – A long unintelligible calculation of nearly 20 pages! Finally obtains relations like

$$\left[\underset{\sim}{g}^{\bar\beta}, \underset{\sim}{g}^{\alpha}\right] = 0 \quad (1)$$

[$\underset{\sim}{f}^{\beta\alpha}$ = scattering matrix, $\underset{\sim}{g}^{\alpha}$ = matrix element of a 'coupling operator' g^{α}]

The bootstrap condn is $\sum_s \underset{\sim}{B}_s^{\bar\alpha\bar\beta} = \sum_s \underset{\sim}{B}_s^{\beta\alpha}$ — (2)

in terms of $\underset{\sim}{g}^{\alpha}$ (the limit is (1)). If one assumed only one meson (i.e 2-body) system states were important ~~[illegible]~~ one would have $\underset{\sim}{R}^{\beta\alpha} = \delta_{\beta\alpha}$ and so "mass-condn) ⟹

$$\underset{\sim}{n}_{\beta\alpha} = \delta_{\beta\alpha}/I \quad (3)$$

For the case where there is no meson production for $\omega \le$ [illegible]μ, (3) →

$$\underset{\sim}{n}_{\beta\alpha} \approx \underset{\sim}{\Lambda}_{\beta\alpha}/I, \text{ where } \underset{\sim}{\Lambda}_{\gamma\beta}\underset{\sim}{\Lambda}_{\beta\alpha} = \underset{\sim}{\Lambda}_{\gamma\alpha} \quad (4)$$

A more compelling argument for (4) is the usual one for coupling not [illegible] – [illegible] order, viz. that they should depend on the basic number of free parameters [This is how the CDD ambiguity is settled, since additional CDD poles introduce additional free parameters into the soln & hence are to be excluded] In weak coupling such 'minimal' Chew–Low solns are the solns of 'minimal' Hamiltonians. In strong coupling the argument is as follows:– In the eqn

$$\underset{\sim}{n}_{\beta\alpha} = (\underset{\sim}{R}^{-1})_{\beta\alpha}\, I^{-1} \quad (5$$

the eigen-values of $\underset{\sim}{R}_{\beta\alpha}$ $\left\{R(\omega) = \sigma/\sigma_{el} \;\&\; R = \frac{2\mu}{\pi}\int^{\infty}\frac{d\omega k}{\omega^3}\left[R(\omega) + \bar R(\omega)\right]\right\}$, ~~$r_\alpha$~~ r_α say, represent parameters which cannot depend on the couplings if we assume that there is only one basic coupling parameter, since it goes to a fixed value viz. ∞, in the strong coupling limit. The r_α are thus either free parameters or they are fixed at some "universal" values. Their possible range of values is $1 \le r_\alpha \le \infty$. Thus we argue that if r_α are not free parameters then they are fixed either at 1 or ∞. Eqn (4) then follows, as well as vanishing of meson production at finite energies.

One may well complain about the preceding argument – that numbers such as π or e or √2 etc are just as much universal as 1 or ∞. Presumably there must exist a better argument for the r_α in the strong coupling limit. [illegible] of course, one may

(*) Condition (b) in this set is that one wants a composite particle to be reasonably similar to an elementary particle, so that one can write down dispersion relations for its scattering & so forth, just for an elementary particle [Cf [illegible] [illegible]]

"cheat" by using Hamiltonian theory; but finally I use "better argument" simply to [illegible] [illegible] [illegible] [illegible] [illegible] in general.

[Ref. to G. C. Wick in Rev. Mod. Phys., 27, 339, 1955]

(13) Ibid – Vol IX A (1967) – Mathematical methods of theoretical Physics,
p. 119 – Field theory of composite particles and bootstraps by M. M. Broido,

I Introduction – Use of quantum field theory partly in the Green's function form developed by J. G. Taylor in this volume – When I talk of field theory I mean the tool used by physicists in discussing quantum electrodynamics, current algebra & so forth – Writes down (b)(*) conditions as desiderata for a field theory of composite particles – I ~~sh~~ will concern myself with giving a careful general account of the notion of composite particle in field theory.

I will define a bootstrap in field theory to be a system of particles each of which is a composite of itself and of the others. This definition has to be justified by showing that systems describing such particles are ones, in appropriate approximations, of the types of bootstrap systems discussed in S-matrix theory. Thus the "bootstrap condition" involved is essentially that of self-consistency. It is not clearly understood (even in S-matrix theory) whether subsidiary bootstrap conditions, such as Levinson's theorem, the absence of subtractions and the existence of uniquely determined sets of masses and coupling constants, are necessary or even consistent with the self-consistency requirement. To conclude, field theory is a very natural method of studying the bootstrap idea. The reason is that bootstraps are expected to be most useful or promising for the discussion of strong interactions, whose high energy processes will be dominated by many-particle intermediate states. Of the various formalisms available to us at the moment, only in field theory do we feel we know (at least in principle) how to handle these.

II Particles – Elementary & composite particles – Physical basis is local Lagrangian theory using renormalisation methods & Green's function – We now define an elementary particle to be a particle whose field cannot be eliminated from the Lagrangian in a local fashion by using the eqns of motion; a composite particle is one which can be eliminated by the use of local functions – Reformulation: A bootstrapped system of particles is one in which each particle field can be

* [R. K. Rockmore Phys. Rev. 112, 878 (1963)]

eliminated from the Lagrangian by the use of [perhaps suitable] quantities arising from the equations of motion

III Making composite particles from elementary ones — Setting up in detail the mechanism by which this is done, using field theory (detailed Green's fn analysis etc) leading to the result: A composite particle in a unitary theory is an elementary particle whose wave function renormalisation constant is set equal to zero — Consideration of some old controversy in quantum electrodynamics — Consideration of a theory of two scalar particles ϕ_a & ϕ_c & trying to make the c-particle composite according to field theory method using renormalisation — Specification in terms of the compositeness criterion: (i) Fix the multiplicative factor in the wave fn normalisation (ii) Express Z_c for the composite particles to be as fn of their desired bound state masses m_c and renormalised coupling constants g_r. (iii) Let $Z_c \to 0$ such that the limit of $\lambda = -\frac{g_1 Z_1}{Z_c \delta m_c^2}$ is finite & $\neq 0$. (iv) If this be not possible for any suitable values of the bound state mass, then there will be no composite particles

IV Bootstraps in field theory. — The above treatment has to be modified as far as bootstraps since they are defined as composite particles of a special kind. The defn means that each particle field can be eliminated from the Lagrangian or that all wave fn renormalisation constants vanish. Now if one tries to consider a simple particle, clearly the defn in terms of elimination from the Lagrangian fails, but one can still fall back on the vanishing of the wave fn renormalisation constant together with the residue condn [(iii) above] on the coupling constant. In order to justify the term "bootstrap", I will have to show that these conditions have something to do with the type of bootstrap eqns which have been discussed in detail in S-matrix theory. I will therefore give brief treatments of the connection with the following four types of bootstraps, which represent a selection from the vast literature [Ref. to Broido & Taylor Phys. Rev. (to appear in July 66) and something else (to appear)]

IV A — The vertex-function bootstrap of Cutkosky [Phys. Rev. 131, 1888 (1963)]

IV B — The N/D type of bootstrap* [~~J. Math. Phys. 7 (1966) (to appear)~~]

IV C — Bootstraps arising from the Mandelstam-structure approach to the Green's fn equations for the $\lambda\phi^4$ theory [Ibid. J. Math. Phys. 6, 1762 (1965)

IV D — Two-particle unitary bootstraps [M. Ida, Progr. Th. Phys. 34, 92, 990 (1965)

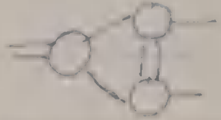
I shall then deal briefly with the following assorted topics

IV E – Lemmas for setting up field theory bootstraps for particles, their [illegible] and other quantum numbers

- IV F – Cut-offs & subtractions in bootstrap theory and their relation to ren- formalism

IV G – Algebraic structure of field-theory bootstraps and how to handle them [Broido, J. Math Phys 7, 1966 (to appear)

Derivation of basic Green's function equation

IV A – By approxn of vertex fn in IV (3) g +  by the constant g_3. The resulting eqn is on the [illegible] mass-shell of the composite, precisely identical to the Cutkosky vertex of bootstrap

[illegible] Feynman [illegible]

IV B – N/D Bootstrap – Following Rockmore, the relationship between the S-matrix theory, self-consistency conditions and the field theory condition

$Z_3 = 0$ [eqn III (15) i.e. $Z_C = \frac{d^2}{dp^2}\, g$ $\Big|_{p^2 = m_C^2}^{-1}$] and

the condition III (16) [$\lambda^{-1} = \lim_{Z_C \to 0} - \frac{{}^2C^5 m_C^2}{g}$ $\Big|_{p^2 = m_C^2}$]

- [illegible] coupling constant, is first established. [uses the elastic unitary cond.n ?, $\mathrm{Im}\, M(s) = M^+(s)\, \frac{k(s)}{8\pi s^{1/2}}\, M(s)$ for s_{AB} (covering two different particles A & B giving rise to a stable neutral spinless composite C) $\equiv (M_A + M_B)^2 \leq s$, where [illegible] is the usual kinematic factor. Then the Chew–Mandelstam N/D form

$$(8\pi)^{-1} M(s) = N(s)\, D^{-1}(s)$$

gives the N/D eqn

$$N(s) = \frac{1}{16\pi i}\int_\gamma \frac{[M(s')\, D(s')\, ds'}{s - s'}$$

$$D(s) = 1 - \frac{1}{\pi}\int_{s_{AB}}^{\infty} \frac{q(s')\, N(s')\, ds'}{(s')^{1/2}\,(s' - s)}$$

-- Another one page work]. Let us consider precisely the implications of these calculations. It is well-known that the Green's function equations we have been using have the same maximal analyticity as S-matrix theory [J. G. Taylor, Phys. Rev. 136, B 1134 (1964)]. Since our approximations have been carried out in a "self-consistent" way, from the point of view of pole structure, we expect this analyticity to persist even in the [illegible] (IV (3)). This analyticity is, of course, enough to give the dispersion relations and to write down the N/D structure. Rockmore's work, which has frequently been misinterpreted, does not claim to show that the N/D eq^ns themselves hold in field theory. This latter assumption must be proved separately as done here. Rockmore's paper also contains a similar self-consistency discussion for vertex-fn bootstrap, (as done in IV a). The use of N/D method for higher partial waves (ie for exchange of particles of higher spin) is bedevilled in S-matrix theory by many subtractions and cut-offs, in field theory by renormalisation problems.

IV. C — 2 C A Bootstrap (Two-particle unitarity without N/D) — We deal with $\lambda\phi^4$ pseudoscalar meson theory, without composite particles, a priori — uses Greens functions & related finite systems of eq^ns using all topologically distinct permutations of the external lines — when $\lambda = 0$, the eq^ns form a Salam-type bootstrap

IV. D. Two particle unitarity bootstraps using N/D — In spite of my criticism of model dependent results (in fact, almost everything in this article is model dependent, my (BSM's) remark), I would like to draw attention to the recent work of Y. Ida, where he makes a detailed analysis & comes up to the classification of particles like 'super-elementary', 'elementary', 'intermediate' and 'composite' (with a table given)

IV E. Cutoffs & subtractions in Bootstrap Theory — Refers to work of Huang & [illegible] very extensive bootstrap calculations in the Chew-Low model [J. Math. Phys, 6, 795 (1965)] — Levinson's theorem & its use in bootstrap theory — Counterexample of Srivastava & Nath — Similar difficulties of principle arise in the recent work of Dashen & Frautschi [Phys. Rev. 143, 1171 & 145, 1287 (1966)] They also have difficulties with subtractions. They give formal arguments suggesting that they can use unsubtracted dispersion relations. However their work appears to contain a determination of the subtraction constant in the pion-nucleon electromagnetic form factor. This is logically equivalent to including the photon in the bootstrap scheme. However this cannot be done in quantum electrodynamics. Of course, it is possible that an S-matrix electrodynamics may one day be written down in which the photon can be exhibited as a composite. Until then, this point will continue to give trouble in any attempt to formulate logical bootstrap theories.

One might also hope that Regge pole effects may arise, (it is not known at the moment how Regge poles can arise out of a general field theory (i.e. without making special approximations involved in, say, iterating a Bethe-Salpeter eqn)). Thus we have a complex of unsolved problems to deal with before relationships between bootstraps & field theory can be really understood.

IV F – Setting up field theory bootstraps – The kinematical terms drop out (putting $Z_3 = 0$) & we have a L of the form

$$\frac{1}{2}\sum_i Z_i\,\delta m_i^2\,\phi_i^2 + \text{interaction term}$$

giving rise to field eqn for each particle

$$Z_i\,\delta m_i\,\phi_i = -\,\text{current}$$

& in each case the unspecified term is exactly what is available if one were dealing with ordinary elementary particles.

IV G. Structure of Field Theory Bootstraps – Considering field theory bootstrap arising from $g\phi^4$ scalar or pseudo-scalar meson theory [g = unrenormalised coupling const], the resulting bootstrap eqn is then

$$\phi = \lambda\phi^3 \qquad (a)$$

(a) means that there is a point-wise multiplication:

$$\phi(x) = \lambda[\phi(x)]^3$$

Where one has to use a Wick product on R.H.S. — discussion of algebraic structure of renormalised Green's fn eqns ~~in~~ and Wick form – this discussion necessary because many people have lingering doubts in the back of their minds as to whether eqn like (a) are real 'maths' or 'not' whatever that may mean — physical interpretation of structure of (a) using algebraic arguments, some kind of 'spectral analysis', boundary condns connected with canonical commutation relations & Haag's theorem.

The results mentioned apply to particles of the given quantum numbers independently of the specific form of the interaction – In both cases, the only form of the theory which seems to survive is the scalar-spinor theory with PCAC derived from

$$\mathcal{L}_I = g\psi\gamma_5\gamma_\mu\psi\partial_\mu\psi$$

giving rise to the bootstrap eqn

$$\psi = \lambda\gamma_5\gamma_\mu\psi\partial_\mu\psi$$
$$\phi = \lambda\partial_\mu(\bar\psi\gamma_5\gamma_\mu\psi)$$

which cannot be handled by our algebraic arguments — Probably the most interesting aspect in composite particles at the moment is the analysis of "bootstrap symmetries"

1968

Note: — In Springer-Verlag article Vol. 45 by Collins & Squires, there is no reference to Broido, Rockmore, Ida or Taylor — I wonder why?

25/1/70) From the Majorana International School of Physics, July 1-14, 1967

p. 393 — Article on all possible symmetries of the S-Matrix by S. Coleman

Discussion — Q. Must I conclude that the Gell-Mann-Okubo mass formula is a gross accident?

Coleman — No. Group theory can tell you a lot, but don't become fanatically so. There is more in the world than algebra; for eg there is dynamics.

Q. How then is your theorem relevant to real physics.

Coleman — It depends on what you mean. Consider isospin symmetry. Everyone believes that this is a good approximate symmetry, but no one believes it to be an exact symmetry of nature. What do I mean by good approximate symmetry? To me this means that the real world doesn't possess this symmetry but there exists an imaginary world which does have this exact symmetry and the predictions in this imaginary world describe well a certain set of events in the real world. For the case of isospin, there is no symmetry generator turning the stable proton into the unstable neutron; this transformation occurs in a hypothetical world in which both particles have the same mass & both are stable.

Q — SU(6) being invalid as a symmetry scheme, but existence of an SU(6) symmetry relationships among coupling constants

Coleman — The Chew-Low theory is a good example. There we connect N* parameters with nucleon (N) parameters. It would be ridiculous to say that because we get a formula relating the mass & coupling constants of the N* and N, there must be an underlying symmetry group, relating the two particles which doesn't have the required direct-product structure of our theorem. Nonsense! The Chew-Low eqns are a product of the dynamics, which is something else again.

Remarks from Gell-Mann's concluding speech p. 693. — Radiative corrections are as ambiguous as ever. We don't know what vertices there are connecting the weak and e.m. interactions, we don't have many ideas of what to do. It is striking that the first radiative correction to the decay of the muon is finite. We do not know what that signifies in view of the fact that we cannot calculate anything higher with the weak interactions. This fact, though, has been used recently by Cabibbo, – and by [illegible] .. in discussing the relation between the weak interaction currents of leptons & of hadrons. If the hadrons ~~were~~ are really organised to look like quarks with the algebraic structure associated with quarks, then we have a problem different from that of the muon. In fact, the case of quarks, where the emission of a lepton pair takes place in going from charge 2/3 to −1/3, is different from that of the muon, where it takes place in going from charge 0 to −1. And here the radiative corrections are finite and there they are not. Since, however, quarks seem to play a useful role in accounting roughly for certain properties of the hadrons, this makes us a little perplexed. We don't know whether to construct some alternative scheme which resembles quarks for describing the symmetry properties of the hadrons or whether we should ~~expe~~ find a compensating term for the ∞'s arising [illegible] in the radiative corrections or perhaps forget the whole problem.

This is just an example of how ignorant we are about the fundamentals.

p. 699 – A lot of theoretical work has been done lately on further structure in the complex plane of A.M. We know about moving poles, we know that moving poles generate moving cuts, ever since Mandelstam's unreadable paper on this subject; anyway besides these there are many other phenomena that people have investigated recently. Using similar methods Stanley Mandelstam has now shown that it is it's quite possible to have fixed poles in the hadron scattering amplitudes; in particular, they are supposed to occur through the same kind of mechanism which predicts the cuts, at a point where a trajectory passes through an integer or half-integer with the wrong signature to make a real particle.

The actual existence of such a pole hasn't been demonstrated, because it isn't ~~hasn't~~ ~~be~~ hasn't been demonstrated yet that its coefficient is actually non-zero; but it could, and it should be done, because at some particular points of the s-variable things can be done exactly, with no approximation. Fixed poles have also appeared in scattering processes involving currents: Compton scattering for eg. In such processes a fixed pole has to show up in order to accommodate the current algebra so that in view of the correctness of some of the aspects of current algebra, fixed poles are already there.

Il Nuovo Cimento 14, 951 (1959) – Regge's paper introducing complex [illegible]

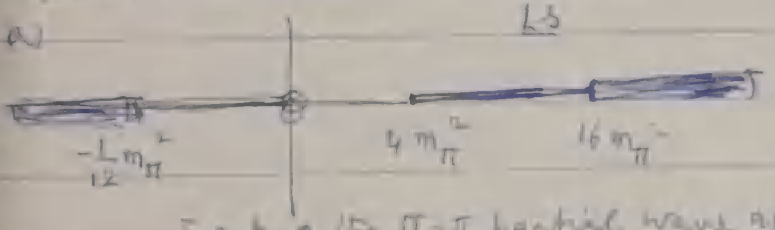
Summary. In this paper the orbital momentum j, until now considered as the integer discrete parameter in the radial Schrödinger eqn is allowed to take complex values. The purpose of such an enlargement is not purely academic, but opens new possibilities in discussing the connection between potentials and the scattering amplitudes. In particular, it is shown that under reasonable assumptions, fulfilled by most field theoretic potentials, the scattering amplitude at some fixed energy determines the potential uniquely, when it exists. Moreover, for special classes of potentials V(x), which are analytically continuable into a function V(z), z = x + iy, regular & suitable bounded in x ≥ 0, the scattering amplitude has the remarkable property of being continuable for arbitrary negative and large cosine of the scattering angle and therefore for arbitrarily large and positive transmitted momentum. The range of validity of the dispersion relations is therefore much enlarged.

Ibid., 18, 947 (1960) – Article by Regge

Summary – In a previous paper a technique involving complex angular momenta was used to prove the Mandelstam representation for potential scattering. One of the results was that the number of subtractions in the transmitted momentum depends critically on the location of the poles (shadow states) of the scattering matrix as a fn of the complex A.M. In this paper the study of the position of the shadow states is carried out in much greater detail. We give also related inequalities concerning bound states & resonances. The physical interpretation of the shadow states is then discussed.

Further notes from the Springer Tract Vol 45.

a)



[Cuts of the π-π partial wave amplitudes

b) i) N/D eqns [Contd from (1) on p. 22]

is given by

$$B_\ell^\pm(s) = \frac{1}{16\pi^2}\int_{t_0}^{\infty} Q_\ell(z')\, D_\ell^\pm(s,t')\,\frac{dt'}{2q_s}$$

P.T.O.

VI.6 — The new form of the strip approximation is distinguished from an earlier form proposed based on the elastic double spectral function. This older form has been revived more recently, however, and will be used in the next section.

Calculations with the new form of the strip approxn have been carried out for π-π scattering with the simplification of equal mass kinematics & zero spin external particles. The method consists in dividing the amplitude into three contributions

$$A(s,t) = A^s(s,t) + A^t(s,t) + A^u(s,t)$$

where the s-channel poles and 2-particle thresholds are contained in the $A^s(s,t)$, the t-channel ones in $A^t(s,t)$ etc. So the potential for generating the s-channel poles is given by

$$V^s(s,t) \equiv A^t(s,t) + A^u(s,t)$$

The approxn consists of writing each term in the form

$$A^t(s,t) = \sum_j A_j^t(s,t)$$

where $A_j^t(s,t)$ is the contribution of the jth t-channel Regge pole and we include in the summation all ~~poles~~ those poles which reach the right-half ℓ_t plane. Thus we are neglecting other types of poles. For each such pole term we require that it include first the one Regge pole & no other poles, though it will contain the various t-channel threshold branch points. It must also be constructed such that

$$A_j^t(s,t) \to 0 \text{ as } t \to \infty .$$

since we expect the s- and u-channel poles to dominate at high t, not t-channel ~~ones~~. And we also require that the amplitude should satisfy the Mandelstam repn, which with this decomposition means that each pole term individually must satisfy the Mandelstam repn. Such a repn called the Chew-Jones repn of a t-channel pole can be easily written down & is ~~known as~~ denoted by $R_j^{s,1}(s,t)$. The complete contribution is then given by

$$A_j^t(s,t) = R_j^{s,1}(s,t) + (-1)^{J} R_j^{u,1}(u,t).$$

where the second term has ~~the~~ its double spectral function bounded by $t = t_1$ & $u = u_1$.

Bergmann's Intn to Relativity.

Part I – Special Theory

Chap. V – Vector & tensor calculus in n-dims

Eqn (5.1) should be $r_{12}^2 = c^2(t_2-t_1)^2 + \{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2\}$

p.67 – Tensor analysis where notion of parallel displacement is introduced

For cartesian coord. system, the law of ||l displacement (ie displacement to a neighbouring point without changing the values of the components) is

$$a_{i,k}\,\delta x_k = 0 \quad (\text{for all } i) \quad (\text{This is obvious})$$

For a general coord. transformation

$$\xi^i = f^i(x_1, .. x_n) \quad (i = 1 .. n)$$

$$a_i = \frac{\partial \xi^r}{\partial x^i}\, a'_r \quad (\text{covariant density vector components of the new system})$$

$\frac{\partial a_i}{\partial x^l}$ = Explanation of the notion of ||l displacement in cartesian systems, obtaining the relation which is a bilinear function of the vector components, and the components of the infinitesimal displacement – hence assumes in general for any ~~other~~ system

$$\left.\begin{aligned} \delta a^i &= -\overset{I}{\Gamma}{}^i_{kl}\, a^k\, \delta\xi^l \\ \text{and } \delta a_k &= +\overset{II}{\Gamma}{}^i_{kl}\, a_i\, \delta\xi^l \end{aligned}\right\}$$

Next obtains the transformation laws for the two Γ's (although left arbitrary) – Derivation of $\overset{I}{\Gamma} = \overset{II}{\Gamma}$ and assumption of symmetry in the indices – Finally unique determination by connecting them with the metric tensor – Differentiation of tensors & covariant derivative.

Use of the Christoffel symbols Γ in finding eqn of geodesic lines.

~~Examples~~: – Considerations relating to special relativity

Examples (1) Prove that the R.H.S of eqn (5.90a) transforms according to eqn (5.83)

$$\text{ie } \begin{Bmatrix} l \\ ik \end{Bmatrix} = \frac{1}{2} g^{ls}\left(g_{is,k} + g_{ks,i} - g_{ik,s}\right)$$

transforms according to

$$\overset{I}{\Gamma}{}'^{k}_{ab} = \frac{\partial \xi'^k}{\partial \xi^l}\left(\frac{\partial \xi^r}{\partial \xi'^a}\frac{\partial \xi^s}{\partial \xi'^b}\overset{I}{\Gamma}{}^l_{rs} + \frac{\partial^2 \xi^l}{\partial \xi'^a\,\partial \xi'^b}\right)$$

(Proof in Eisenhart)

Q.1. about $\begin{Bmatrix} l \\ ik \end{Bmatrix}$ & $\{l, ik\}$ & above trn & to show that $\{\ \}$ are not tensors

20/3/70 – Handbuch der Physik, Vol IV, 1962 – Article on general relativity by P. G. Bergmann (Lots of new things over & above what is contained in his old book) p.212. Remarks (1) There are certain covariant derivatives and combinations of derivatives, which reduce to ordinary derivatives. Among the most important are the antisymmetrized derivatives of a covariant vector field and of a covariant skew-symmetric tensor field viz.

$$A_{\mu;\nu} - A_{\nu;\mu} = A_{\mu,\nu} - A_{\nu,\mu} \qquad (1)$$

$$A_{[\rho\sigma];\tau} + A_{[\sigma\tau];\rho} + A_{[\tau\rho];\sigma} = A_{[\rho\sigma],\tau} + A_{[\sigma\tau],\rho} + A_{[\tau\rho],\sigma} \qquad (2)$$

p.213 - Curvature tensor $R_{\iota\kappa\lambda}^{\ \ \ \ \nu} \equiv \Gamma^{\nu}_{\lambda\iota,\kappa} - \Gamma^{\nu}_{\lambda\kappa,\iota} - \Gamma^{\nu}_{\rho\iota}\Gamma^{\rho}_{\lambda\kappa} + \Gamma^{\nu}_{\rho\kappa}\Gamma^{\rho}_{\lambda\iota}$ —(3)

Properties of the R.C. tensor.

(i) For general case of curvature tensor

$$R_{\iota\kappa} + R_{\kappa\iota} \equiv 0 \qquad (4)$$

It also satisfies Jacobi relations viz

$$R_{\iota\kappa,\lambda} + R_{\kappa\lambda,\iota} + R_{\lambda\iota,\kappa} + [\Gamma_\lambda, R_{\iota\kappa}] + [\Gamma_\iota, R_{\kappa\lambda}] + [\Gamma_\kappa, R_{\lambda\iota}] = 0 \qquad (5)$$

(ii) Bianchi identities when affine connection is symmetric viz.

$$R_{\iota\kappa\rho;\lambda}^{\ \ \ \ \sigma} + R_{\kappa\lambda\rho;\iota}^{\ \ \ \ \sigma} + R_{\lambda\iota\rho;\kappa}^{\ \ \ \ \sigma} \equiv 0 \qquad (6)$$

& in addition the algebraic identities

$$R_{\iota\kappa\lambda\cdot}^{\ \ \ \ \nu} + R_{\kappa\lambda\iota\cdot}^{\ \ \ \ \nu} + R_{\lambda\iota\kappa\cdot}^{\ \ \ \ \nu} \equiv 0 \qquad (7)$$

(iii) If affine connection consists of Christoffel symbols

$$R_{\iota\kappa\lambda\mu} + R_{\iota\kappa\mu\lambda} \equiv 0 \qquad (8)$$

where $R_{\iota\kappa\lambda\mu} \equiv g_{\mu\nu} R_{\iota\kappa\lambda}^{\ \ \ \ \nu}$

(iv) contracted forms: Contracting $R_{\iota\kappa\lambda}^{\ \ \ \ \nu}$ on the 1st & last index, we obtain the so called Ricci tensor

$$R_{\kappa\lambda} = R_{\sigma\kappa\lambda\cdot}^{\ \ \ \ \sigma}\ ; \quad R_{\kappa\lambda} = R_{\lambda\kappa} \qquad (9)$$

If we contract once more we get the curvature scalar R

$$R = g^{\kappa\lambda} R_{\kappa\lambda} \qquad (10)$$

The combination $G_{\kappa\lambda} \equiv R_{\kappa\lambda} - \frac{1}{2} g_{\kappa\lambda} R$ (11)

is called the Einstein tensor.

The Ricci & Einstein tensor are both symmetric. The Einstein tensor

[covariant derivative defined by $V^{\mu}_{;\rho} \equiv V^{\mu}_{,\rho} + \Gamma^{\mu}_{\alpha\rho} V^{\alpha}$; $v_{\mu;\rho} \equiv v_{\mu,\rho} - \Gamma^{\alpha}_{\mu\rho} v_{\alpha}$]

has the further property that its covariant derivative vanishes identically, a fact that can be readily ~~seen~~ derived from ~~the~~ Bianchi identities (6) by contracting them on two separate index pairs. Hence the identities

$$G^{\kappa\rho}{}_{;\rho} \equiv 0 \qquad (12)$$

are often called the contracted Bianchi identities

(v) No. of algebraically ind. components :

$N_0 = \frac{1}{12} n^2(n^2-1)$ For $n = 2$, $N_0 = 1$ comp, $n = 3$, $N_0 = 6$ for $n = 4$, $N_0 = 20$

The number of components of the Ricci tensor or of the Einstein tensor is 6 in a 3-dimnl space and 10 in a 4-dimnl space. Four is, accordingly, the lowest number of dimensions of a space in which the full curvature tensor contains a greater amount of information about the structure of the space than either contracted form.

(vi) Commutator of covariant differentiation :- For symmetric affine connections the commutator of two covariant derivatives is

$$\left.\begin{aligned} V^{\nu}{}_{;\iota\kappa} - V^{\nu}{}_{;\kappa\iota} &\equiv R^{\nu}{}_{\iota\kappa\lambda} V^{\lambda} \\ V_{\lambda;\iota\kappa} - V_{\lambda;\kappa\iota} &\equiv R^{\nu}{}_{\iota\lambda\kappa} V_{\nu} \end{aligned}\right\} \qquad (13)$$

p. 220 : Weyl tensor $C_{\iota\kappa\lambda\mu}$:

$$C_{\iota\kappa\lambda\mu} \equiv R_{\iota\kappa\lambda\mu} + \frac{1}{2}\left(g_{\iota\lambda} R_{\kappa\mu} + g_{\kappa\mu} R_{\iota\lambda} - g_{\kappa\lambda} R_{\iota\mu} - g_{\iota\mu} R_{\kappa\lambda}\right)$$

$$+ \frac{1}{6}\left(g_{\kappa\lambda} g_{\iota\mu} - g_{\iota\lambda} g_{\kappa\mu}\right) R$$

$$C_{\kappa\lambda} \equiv g^{\iota\mu} C_{\iota\kappa\lambda\mu} \equiv 0.$$

Although this tensor is of rank 4, because of its many algebraic symmetries it has only 10 algebraically independent components.

References & others

1) Rev. Mod. Phys. 21 . No. 3 (1949) – Collection of papers in honour of Einstein's 70th birthday

(2) Helv. Phys. Acta. Suppl. 4 (1956) – Proc. Int. Conf. on Rel.

(3) Rev. Mod. Phys. 29 . No. 3 (1957)

(4) Lectures on General Relativity by Trautman, King's College, London, 1958

23/3/70 Again notes from Bergmann's article in Handbuch d. Physik

Correspondence with Newtonian theory, p 216

Let the fundamental eqⁿ be

$$\frac{dU^{\mu}}{d\tau} + \left\{ \begin{matrix} \mu \\ \alpha\beta \end{matrix} \right\} U^{\alpha} U^{\beta} = 0 , \quad U^{\mu} = \frac{dx^{\mu}}{d\tau} , \quad g_{\mu\nu} U^{\mu} U^{\nu} = 1$$

on the strength of the principle of equivalence. Using "linearisation approxⁿ" & a "slow approximation" reduces above eq the in the Newtonian case to

$$\frac{d\underline{u}}{d\tau} + \frac{1}{2} \nabla^{2} g_{00} = 0.$$

Justifies this on explicit basis, thereby showing that by approxⁿ of field eqⁿˢ can be reduced to Newtons law of gravitation

The actual reduction given here is quite complicated as in Bergmann's text-book.

p. 230 - In one of the several forms to be found in the literature, Schwarzschild's line element takes the form

$$g_{\mu\nu} = \begin{pmatrix} -\left(\delta_{mn} + \frac{R}{r} \cdot \frac{x^{m} x^{n} / r^{2}}{1 - R/r} \right. , & 0 \\ 0 , & 1 - R/r \end{pmatrix}$$

$r^{2} = (x^{1})^{2} + (x^{2})^{2} + (x^{3})^{2}$, $R = 2\kappa\mu$ [κ = gr. const ?
μ = mass of particle]

Constant R is known as the Schwarzschild radius of mass μ. All line elements static or otherwise, that have spherical symmetry are coordinate transformations of this solution (Birkhoff's theorem). Incidentally this theorem implies that a spherically oscillating source of the gravitational field will not produce outgoing gravitational radiation

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P.C. Vaidya's lectures at Matscience (Report 49)

p. 3 — mentions Newton's theory as → weak fields & small velocities.

p. 10 – because Schwarzschild's soln is static & have no strictly spherical waves.

p. 36. – Schwarzschild singularity – For a sph. sym. ~~soln~~ distn of matter, we have, outside the matter, we have S's soln of the eqⁿ

$$R_{ij} = 0$$

given by $ds^2 = \left(1-\frac{2m}{r}\right)dT^2 - \left(1-\frac{2m}{r}\right)^{-1}dr^2 - r^2 d\Omega^2$

As it is apparent, this ~~bas~~ sol^n has two singularities one at $r = 0$ & another at $r = 2m$.
The surface $r = 2m$ is a null surface. Can we transform this singularity? Choose a time coord. u such that

$$\frac{\partial u}{\partial r}\left(1-\frac{2m}{r}\right)^{1/2} + \frac{\partial u}{\partial t}\left(1-\frac{2m}{r}\right)^{-1/2} = 0$$

Clearly u is the retarded time. The transformation of T to retarded time u leads to the following line-element

$$ds^2 = \left(1-\frac{2m}{r}\right)du^2 + 2\,du\,dr - r^2 d\Omega^2$$

Apparently $r = 2m$ is not a singularity here. To see this more clearly, consider the tr^n $u = t - r$. This makes

$$ds^2 = \left(1-\frac{2m}{r}\right)dt^2 + \frac{4m}{r}\,dr\,dt - \left(1+\frac{2m}{r}\right)dr^2 - r^2 d\Omega^2$$

$$ds^2 = -dx^2 - dy^2 - dz^2 + dt^2 - \frac{2m}{r}(dt - dr)^2$$

This clearly shows that $r = 0$ is the only singularity of our sol^n. This line-element was first discovered by Eddington & later by Finkelstein.

Since we know from gen. rel. that any set of tr^{n}s to diff. systems do not alter the physical situation (principle of covariance), obviously the singularity $r = 2m$ could not have escaped into nothing. By regarding the transformation from T to t directly we find that it is given by

$$T = t \pm 2m \log(r - 2m)$$

Apparently we have the singularity in the tr^n eq^n.

Now of the two, the Schwarzschild & Finkelstein sol^{n}s which represents the physical situation? For $r > 2m$, the S-sol^n → to F-sol^n by the tr^n $T = t \pm 2m\log(r - 2m)$. Hence in this case we have t-axis as time-like vector & sol^n obtained is static

For $r < 2m$, the tr^n will have to be

$$T = t \pm 2m\log(2m - r)$$

& we find when we go over to the S-metric roles of t & r will have interchanged ie coeffts involve t & sol^n is not static

Physical properties of surface $r = 2m$

(1) If an object with $r < 2m$ emits radiation it is not observable ∵ at $r = 2m$ we get zero-red.

(2) For object $r > 2m$, it will contract to the S-radius in ∞-time

From energy considerations, M (S-mass) is the max. mass in radius R or R = 2M

- is the min. radius for given M. Thus objects with R ≤ 2M have no physical significance

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Synge – Gen. Relativity –

Tolman, p. 198 – Newton's theory as 1st approxn – Improved metric

take $g_{11} \simeq g_{22} \simeq g_{33} \simeq -1, \; g_{44} \simeq 1$

$g_{\mu\nu} \simeq 0 \quad (\mu \neq \nu$

and $\frac{g_{\mu\nu}}{\partial x^4} = 0$. & also $\frac{dx^1}{ds} \simeq \frac{dx^2}{ds} \simeq \frac{dx^3}{ds} \simeq 0, \; \frac{dx^4}{ds} \simeq 1$.

(a) Motion of free particle in gravitational field

$$\frac{d^2x^\sigma}{ds^2} + \{\mu\nu, \sigma\}\frac{dx^\mu}{ds}\frac{dx^\nu}{ds} = 0$$

reduces to $\frac{d^2x^\sigma}{d(x^4)^2} + \{44, \sigma\} = 0 \qquad (\sigma = 1, 2, 3$

or $\frac{d^2x^\sigma}{d(x^4)^2} = -\frac{1}{2}\frac{\partial g_{44}}{\partial x^\sigma}$ — (i)

Defining ~~ψ/c^2~~ Newtonian potential ψ by $\psi/c^2 = g_{44}/2 + \text{const}$

- or $g_{44} = 1 + 2\psi/c^2$ (2), (i) reduces to $\frac{d^2x}{dt^2} = -\frac{\partial\psi}{\partial x}$, —, —

(b) Poisson's eqn as an approxn for Einstein's field eqs – to complete above justification of ψ as Newtonian potential, we must also show E-eqn → N-eqn. Using eqn for

$T^{\mu\nu} = p_{xx} \quad p_{xy} \quad p_{xz} \quad cq_x$

— — $\qquad g$ = momentum density!

$p_{zx} \quad - \quad p_{zz} \quad cq_z$

$cq_x \quad cq_y \quad cq_z \quad c^2\rho$

We see in this approxn all components except $T^{44} = c^2\rho$ are zero provided we have a weak field & p_{ij} are negligible cf. density of energy. We can also write $T^{44} = T^4_4 = T_{44} = T = c^2\rho$. Combining this with

$$-\kappa T_{\mu\nu} = R_{\mu\nu} - \tfrac{1}{2}Rg_{\mu\nu} \; ; \; \kappa T = R$$

we have $-\kappa c^2\rho = R_{44} - \frac{1}{2}\kappa c^2\rho\, g_{44}$

or $R_{44} = -\frac{1}{2}\kappa c^2\rho$ — ~~(2)~~ (3)

~~Using both~~ neglecting products of 3-index symbols (∵ of weak field) & $\frac{\partial}{\partial t}$'s are zero (∵ of static field) (1) reduces to

$$\frac{\partial}{\partial x^\sigma}\{44, \sigma\} = \tfrac{1}{2}\kappa c^2\rho$$

and using (2) in (a), $\frac{\partial^2}{\partial x^2}\left(\frac{g_{44}}{2}\right) + \ldots + \ldots = \frac{K\sigma^2}{2}$ —(3)

& if we again take $\psi/c^2 = \frac{g_{44}}{2}$ + const as in (a) and assign to the value $\kappa = 8\pi K/c^4$ (K = ordinary constant of gravitation), (3) is exactly Poisson's eqn in Newtonian theory of gravitation

(15|4|70) — Phys. Review (25|11|69) (Vol. 187, No. 5)

p. 1811 — Non-uniqueness of the spin-½ equation by Anton Z. Capri
Ref. to Shattis article in Rev. Mod. Phys. (No ref. to my work).
— Also refers to Takahashi at the end for thanksgiving.

must be looked into

The American Scientist — Winter 1969 (Vol 57, No. 4)
p. 517 — The fourth force in nature by R. E. Marshak — Treats of weak interaction forces in popular style

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(20|4|70) — New Book Section — The Study of the history of mathematics and the study of the history of Science by G. Sarton (Dover Publ. N.Y. — 2 Vols. bound as one — No reference to Hindu Mathematics at all! Only ref. to Arabs.

Science — Recent Vol. 160, 10|5|1968 — pp. (664-665) — Views of Schrödinger, Einstein etc. on "is quantum mechanics" — Ref. to Born as having already given in 1924 the name "quantum mechanics" to the wave mechanics or matrix mechanics". — Also other references to Born.

Vol. 160 (17|5|68), p. 742 — Mathematics for Biologists, Chemists and Physicists — Recommendation of the NSMRSE (National Study of Maths for Scientists & Engineers) by G. H. Miller.

p. 736 — Solid State Physics as a source of modern Electronics. by Lawrence A. Hall

~~Vol~~ Ibid. (24|5|68), p. 837 — Letter re. how a good student in mathematics was handicapped by a so-called "modern" text book on Calculus.

Ibid. (7|6|68) — Editorial on "Matching Education & jobs in developing nations".

p. 1102 – Review of Timoshenko's autobiography by J. P. den Hartog – First in U.S.S.R, then in U.S.A, & still living at 90 hale & hearty – Interesting memoir named "The Odessey of an Engineer".

Ibid (14/6/68); p. 1209 – Max Planck Society* – Filling a gap in German Research – Interesting article

[* Called MPG (Max-Planck-Gesselschaft fur Forderung der Wissenschaften)]

Ibid (21/6/68) p. 1295 – Article by Mc Crea on "Cosmology after half a century"

(22/4/70) Newbook Section – (1) Proceedings of 13th Conference on the structure and evolution of galaxies.

– Host of interesting papers & discussions.

p. 164 – Remark by W. A. Fowler :– Energy generation in stars by means of exothermic nuclear reactions has stimulated many interesting and fruitful researches in nuclear physics. In approaching these new problems we are accustomed to begin with Einstein's mass-energy relation $E = Mc^2$. However, this can be quite misleading. It is better to write

$$E = -E_B = (M - M_0)c^2 = 2 \times 10^{54} (M - M_0)/M_\odot \text{ ergs}$$

where E is the energy (exclusive of the rest mass energy) of a system composed of particles with total rest mass M_0 and E_B the +ve binding energy of the system.

p. 168 – Interesting general discussion of topics of interest about which we know very little.

(2) Enrico Fermi Series – XXXI Corso – Quantum Electronics and Coherent light

p. 284 – Stimulated Raman effect by A. Javan.
p. 306 – Stimulated Raman scattering by B. P. Stoicheff
p. 326 – Raman and Phonon masers by 3 authors

(3) Annual Review of nuclear science edited by Segre et als - Vol 19, 1969

✓ || p. 367 - Parity & time-reversal in nuclear physics by E. M. Henley.

(4) A book on Laser (166 references).

- Springer Verlag, 1969.

(24/4/70) For Anthropaper:

Ref to - Elements of Probability theory by H. Cramer (John Wiley & Sons)

Chaps 1, 2, 3 and 4 only.

For the topics :- Addition and multiplication rules, conditional probability, Baye's rule and simple problems on these results, Tchebycheff's inequality.

Ex. Show that for any distribution $\alpha_4 > 1$, and the quartile coefficient of skewness varies from -1 to $+1$

[Here $\alpha_4 = \frac{\mu_4}{\mu_2^2} = \frac{\mu_4}{\sigma^4}$, μ_2 & μ_4 are moments about the mean

& $\mu_2 = \sigma^2$ = variance

Quartile coeff of skewness $= \frac{Q_3 + Q_1 - M}{(Q_3 - Q_1)}$, M = median

& Q_1 and Q_3 are the first and third quartiles]

Cramer's Book (Found it in the Text books Section)

Q.(a)(i) Explain the addition and multiplication rules of probability, and derive Bayes' theorem from the latter [C. pp. 31, 36, § 3.6, pp. 38-39.

(b) Two cards are drawn simultaneously from the same set of 52 cards. Find the probability that at least one of them will be a heart.

[Unworked Ex. 3, p. 40 of Cramer

Solⁿ :- The probability that both cards drawn are hearts is given by the multiplication rule $P(AB) = P(B)\,P(A|B)$.

Here $P(A) = P(B) = 13/52$ and $P(B|A) = 12/51$.

Hence $P(AB) = \frac{13}{52} \cdot \frac{12}{51} = \frac{1}{17}$.

Next

By the addition rule $P(A+B) = P(A) + P(B) - P(AB)$

$= 13/52 + 13/52 - 1/17 = \frac{1}{2} - \frac{1}{17} = \frac{15}{34}$]

Note - Tchebycheff's theorem is found in Chap 5, ~~p 80~~ p. 81 of Graves - hence is not in the 1st four Chapters.

4/5/70 In the new books section

(1) Elementary particles & High energy Physics - Contains an article by M. Gourdin on symmetries

(2) Group theory & its application to molecular structure by 2 authors

(3) Quark model by J. J. Kokkedee (Univ. of Nijmegen, The Netherlands, W. A. Benjamin Inc. N. Y. Amsterdam, 1969)

(4) The Story of Jodrell Bank by Bernard Lovell - Oxford Univ. Press 1968 - An authoritative account by the founder of the Observatory himself

11/5/70 In the New Books Section

✓✓ (1) Einstein Spaces by A. Z. Petrov (Professor of Relativity and Gravitation Kazan University) - Pergamon Press.

Chap 1 - Basic tensor analysis
Chap 2 - Einstein spaces
Chap 3 General Classification of gravitation fields
" 4 - Motions in empty space
" 5 - Classification of gravi. fields by groups of motions
" 6 - Conformal mapping of Einstein spaces
" 7 - Geodesic mapping of gravitational fields
" 8 - Cauchy problem for the Einstein field eqns
" 9 - Special types of gravitational fields.

(2) The Evolution of Stars - No. of people including Shapley

(3) Topological dynamics - An international Symposium - All sorts of articles.

(4) Observational Cosmology - T. I. F. R. Publn - D. W. Sciama

21/5/70 New Books Section

✓ (1) Book by F. J. Dyson on Symmetry groups in nuclear and particle physics - Math. Phys. Monograph Series - Benjamin (1966)

other books in that case (a) Lie Groups for Physicists by Robert Hermann (b) The Math. foundations of Quant. Mech by R. G. Newton, (C) PCT, Spin, and Statistics and all that by R. F. Streater and Wightman

(2) A book on muons by A. O. Weissenberg — North-Holland Publishers (1967)

Chap 1 :– Properties of muons
" 2 : Muon decay
3 : Weak muon capture
4 : E.M. interactions of muons
5 : Mesomolecular phenomena
6 : Depolarisation of muons, Muonium
7 : Cosmic ray muons

Also refers to a book by Marshak on "Meson Physics"

22/5/70 — New Books Section

(1) Quasilinearization and invariant imbedding by E. Stanley Lee — Academic Press (1968) — with applns to Chemical Engineering and adaptive control

(2) Selected Lectures of the Royal Society — Vol I — Academic Press (1967) — Reprinted from Proc. Roy. Soc.

— An interesting lecture on Rutherford by Kapitza.

25/5/70 — New Books Section

(1) A path to modern mathematics by W. W. Sawyer (Pelican original Introducing Maths. 4) — small book of 221 pages — Contents: 1. Intro

(1) Arithmetic of algebra — 2. A geometrical dictionary — 3. On maps & matrices — 4. Hidden simplicity — 5. Benefits from equations — 6. Invariants applications — 7. Invariants systematic classification — [illegible] 8. On linearity — 9. What is a Rotation — 10. Metric and Banach spaces — Answers

(2) Spectroscopic and group theoretical methods in Physics — Racah Commemoration Volume — North-Holland (Racah was father of theoretical Physics in Israel) (1968)

— Articles by number of people including one from Jaipur on p. 131.

(3) Physics of nuclei and particles Vol I (huge volume) by Pierre Marmier & Eric ~~Sheldon~~ Sheldon — Academic Press (1969) — all sorts of topics — 2 references to Bhabha, but no reference to my work at all in the bibliography.

J. of Math & Mechanics 18, No. 9 March, 1969 (in the collection, not bound in shelf 57 on Ground floor

p. ~~893~~ p. 881. Degenerate gravitational fields with twisting rays by

Ivor Robinson, Joanna R. Robinson, & Joseph D. Zund — This [illegible] looks a
[illegible] — in what I can understand some work on general relativity — Anyway it took
of carefully [illegible] — on p. 892 at the end the article is given a

1. Note added in proof. Václav Hlavatý died on 11 January 1969. This paper
Note is dedicated affectionately to his memory — R. S. Mishra has done a lot of
work with Hlavatý.